Spaceborne Earth Applications Ranging System (SPEAR)

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A technique is discussed for the accurate (i.e., to within fractions of centimeters per year) detection of Earth surface motions utilizing the latest space technology. It is shown that, over a six-day period and assuming a 50% cloud cover (i.e., as experienced over the past few years of laser operation) utilizing spaceborne precision ranging systems, intersite distances on the order of 5 to 15 km (dependent mostly on the beam width of the laser) can be determined in the vertical and horizontal components, with errors in the 0.5- to 1.5-cm range. These errors are almost independent of ground survey errors up to 0.25 m and orbit errors up to 200 m. A spaceborne laser ranging system is assumed to range to two or more ground-emplaced retroreflectors. This can be done either in a simultaneous or nonsimultaneous mode. Hardware is under development for the latter technique.

Introduction

ACCURATE determination of intersite distances has been pursued for many years. Continuous improvement of ground-based laser ranging systems now permits the determination of intersite distances of 1000 km to within tens of centimeters. As early as 1968, an inverting of the system by placing corner cubes on the ground and a laser system in orbit was suggested. By 1971, serious consideration of such a system had begun. At present, work is proceeding on the design of a precision laser ranging system for the space shuttle laboratory.

A major impetus for the development of such a system is provided by its possible application to earthquake prediction. Scholz et al.[1] presented a paper in 1973 stating that dilatancy, among other phenomena, is a real precursor to earthquakes. As outlined in their paper, the ground rises around an active zone. A well-documented case is, to quote the example used by Scholz, the 1964 Niigata earthquake ($m \equiv 7.5$, where $m \equiv$ earthquake magnitude as measured on the Richter scale) (see Fig. 1). Measurements indicated rather clearly a constant vertical motion of the ground of about 12 cm over 60 yr (0.2 cm/yr). Subsequently, the motion stops for 3 to 4 yr until the earthquake occurs. A system as described here could perform such measurements and determine if, in fact, the speculated link between dilatancy effects and earthquakes is real. Accurate (centimeter-level) relative intersite distances are needed in many other practical applications. Such a system, furthermore, could be installed to monitor small motions near or on large dams, major construction sites, shore facilities and structures, etc.

Previous investigations[2-4] have concentrated on purely geometric techniques for estimating intersite distances from slant range observations between an ensemble of ground stations and an orbiting satellite. The essence of these techniques is that the satellite position at all of the observation times are estimated simultaneously along with intersite distances. It is shown in Ref. 4 that if at least six stations are included in the system there is sufficient observability in the data to permit a stable determination of the intersite distances.

The strength of the purely geometric approach is its simplicity. Since the position of the satellite is estimated at each observation time, it is not necessary to model the satellite dynamics or to provide independent tracking support beyond what is required by ordinary stationkeeping. But this approach does not yield an optimal solution, since it does not provide for the utilization of orbit information obtained from independent tracking. This paper provides a theoretically optimal technique for estimating intersite distances from range difference observations. Our approach is computationally more elaborate than purely geometric methods, since it requires the integration of satellite equations of motion. However, it has the advantage of providing the most accurate estimate of intersite distances based on all available information.

Succeeding sections describe a range differencing system and provide mathematical models for optimally estimating intersite distance from range difference observations. The paper also includes the results of computer simulations, which suggest the accuracies that may be obtainable from such a system.

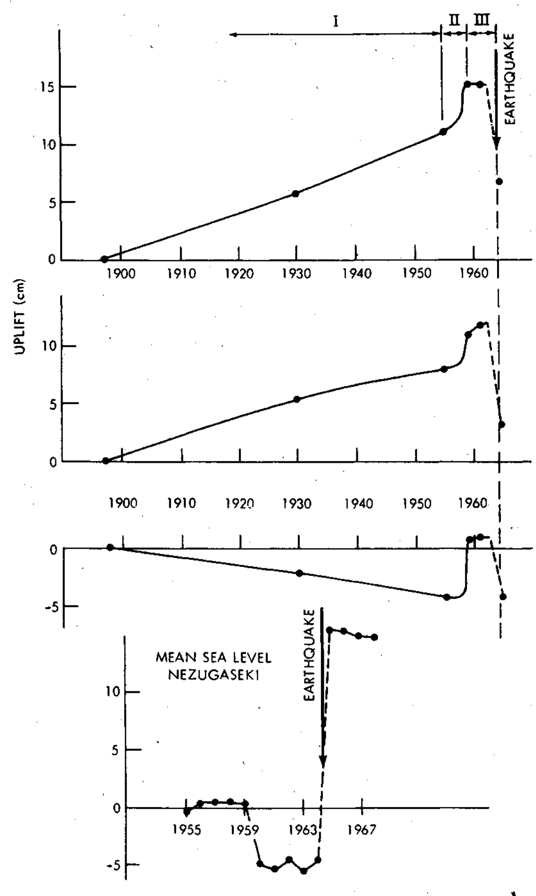

Fig. 1 Observed dilitancy preceding earthquake.[1]

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Systems Concept

As was shown in Ref. 1, the region of rock dilatancy extends one fault length (L) on each side of an active fault zone (total length $=3L$) and one length on each side (width $=2L$). Thus, if one wishes to observe the rise of the ground, an array of ground transponder cubes which covers this active area must be used. Such an arrangement is shown schematically in Fig. 2.

The smallness of the motions to be observed over several years requires the use of a rather specific ranging and/or tracking systems concept. In essence, one must determine very accurately the changes in $|\boldsymbol{D}|$ between two ground points using satellite technology. In general, there are three major error sources to overcome if one wishes to compute vector distances of 5 to 20 km at the 0.5- to 1.5 cm accuracy level. These are 1) orbital errors of the spacecraft, 2) bias errors in the ranging system, and 3) atmospheric propagation errors.

To some extent, the effect of orbit errors can be limited by a range differencing approach. In addition, one can model the dynamics of spacecraft motion and estimate a satellite epoch state for each pass, along with intersite distances. This approach has the advantage of permitting the optimal use of a priori satellite epoch state estimates obtained from independent tracking. The mathematics of this procedure is described in the next section.

Error sources 2 and 3 can be eliminated to a first order by using range differencing. Practically, this implies the measuring of the difference in times of arrival of a pulse from a spacecraft to two or more ground stations; the distance $|\boldsymbol{D}|$ between neighboring ground stations (i.e., transponders or laser corner cubes) can be computed from such observations. However, the use of one pulse to cover more than one corner cube (or transponder) raises a power problem. From strictly a signal-to-noise vantage point, it is preferable to send a single beam at a time to a single ground station. But this approach reintroduces errors 2 and 3, as previously defined. Thus the single-pulse approach is a requirement in eliminating significant error sources in the determination of $|\boldsymbol{D}|$, and the furnishing of sufficient power becomes an important systems design factor. An alternative nonsimultaneous ranging approach, which requires less power, also is under investigation.

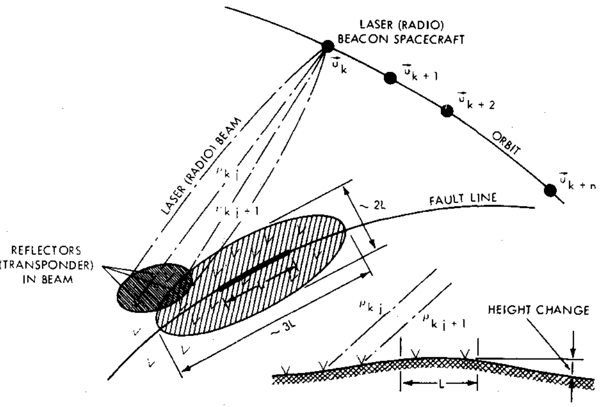

Fig. 2 Rock dilitancy determination from space.

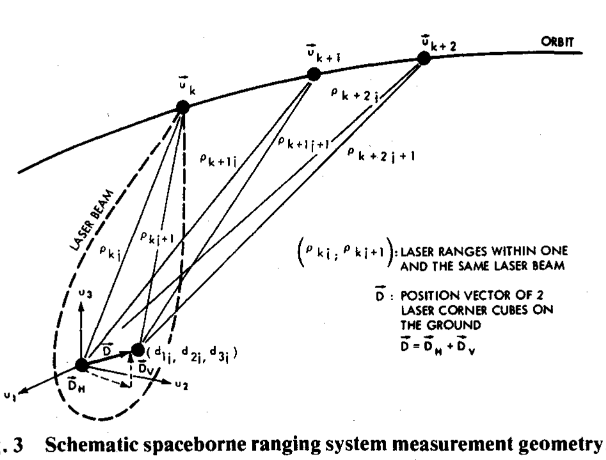

Fig. 3 Schematic spaceborne ranging system measurement geometry.

Intersite Distance Estimation Using Simultaneous Ranging

In what follows, all vectors are referenced to a common geocentric, Earth-fixed coordinate set. Also, it is assumed that relatively small effects, such as polar motions and Earth and ocean tides, are modeled appropriately. S_1 and S_2 represent the vector positions of two corner cubes (or transponders). Suppose that the data are obtained from N satellite passes, where X_i, $i=1, 2,\cdots,N$, is a six-dimensional epoch state for the ith pass. The position of the satellite during the ith pass at time T can be expressed as

$$u_i(T)=U(X_i,T) \tag{1}$$

where the function U is obtained by integrating satellite equations of motion with boundary conditions provided by X_i from epoch time to T. The measurement of a differential time of arrival of a pulse sent from the satellite at time T when scaled by the speed of light is essentially a measurement of the difference of slant ranges. Hence, our fundamental measurement is considered to be (see Fig. 3)

$$\rho_T=[u_i'(T)u_i(T)+S_1'S_1-2S_1'u_i(T)]^{1/2} - [u_i'(T)u_i(T)+S_2'S_2-2S_2'u_i(T)]^{1/2} \tag{2}$$

where the observation is obtained during the ith pass. Arrange all of the observations into a column vector:

$$\boldsymbol{\rho}=\begin{bmatrix}\rho_1\\ \rho_2\\ \vdots\\ \rho_m\end{bmatrix} \tag{3}$$

From Eqs. (1) and (2), it is clear that $\boldsymbol{\rho}$ can be represented functionally as

$$\boldsymbol{\rho}=f(\boldsymbol{Z}) \tag{4}$$

where $\boldsymbol{Z}$ is a $6N+6$ dimensional parameter vector, represented as

$$\boldsymbol{Z}=\begin{bmatrix}X_1\\ X_2\\ \vdots\\ X_n\\ S_1\\ S_2\end{bmatrix} \tag{5}$$

The object is to extract from the observation set $\boldsymbol{\rho}$ in some sense a "best" estimate of the parameter set $\boldsymbol{Z}$. We assume that the range difference approach eliminates biases in the data type. There remains, however, a white noise component to the data. Assume

$$\boldsymbol{\rho}=\tilde{\boldsymbol{\rho}}+\boldsymbol{\nu} \qquad E(\boldsymbol{\nu})=0 \qquad E(\boldsymbol{\nu}\boldsymbol{\nu}')=Q \tag{6}$$

where $\boldsymbol{\rho}$ is a vector of the correct or noiseless values of the observations and $\boldsymbol{\nu}$ is a random vector representing the data noise. Our final estimate of $\boldsymbol{Z}$ must rely on all available information. Hence we must consider the fact that a priori estimates of the epoch states X_i, $i=1,\cdots N$, are available from independent tracking data. Also, a priori estimates of S_1 and S_2 also are available. Let $\boldsymbol{Z_0}$ represent an a priori estimate of $\boldsymbol{Z}$. Then

$$Z_0=Z+\alpha \qquad E(\alpha)=0 \qquad E(\alpha\alpha')=P_0 \tag{7}$$

where α is a random vector representing the errors of the a priori estimate. The covariance matrix P_0 of α assumes the form

$$P_0 = \begin{bmatrix} P_x & 0 \\ 0 & P_s \end{bmatrix} \quad (8)$$

where P_x is the covariance matrix associated with the a priori estimates of the epoch vectors, and where P_s is the covariance matrix associated with the a priori estimates of S_1 and S_2.

Assuming the linearity of Eq. (4), the minimum variance estimator of Z is the estimator that minimizes the quadractic loss function.

$$L(Z) = (\rho - f(Z))' Q^{-1} (\rho - f(Z)) + (Z_0 - Z)' P_0^{-1} (Z_0 - Z) \quad (9)$$

A first approximation to the desired minimum can be obtained by expanding Eq. (4) in a first-order Taylor series about a nominal value Zn of Z

$$\delta\rho = A\delta Z \quad A = \left.\frac{\partial f(Z)}{\partial Z}\right|_{Z_n} \quad (10)$$

where $\delta\rho$ and δZ are the deviations of e and Z from nominal values, and A is the so-called sensitivity matrix. The estimate of δZ is

$$\delta Z = (A' Q^{-1} A + P_0^{-1})^{-1} (A' Q^{-1} \delta\rho + P_0^{-1} \delta Z_0) \quad (11)$$

where

$$\delta Z_0 = Z_0 - Z_n \quad (11a)$$

The left side of Eq. (11) is added to the nominal to form an estimate of Z. This value can be used as a new nominal, and the process can be repeated until a convergence criterion is satisfied.

Again assuming the linearity of Eq. (4), the covariance matrix of the resulting estimator Z is

$$\mathrm{cov}(\hat{Z}) = (A' Q^{-1} A + P_0^{-1})^{-1} \quad (12)$$

Attention is focused on the estimates $\hat{S}_1$ and $\hat{S}_2$ of S_1 and S_2. Since $\hat{S}_1$ and $\hat{S}_2$ are the last six elements of $\hat{Z}$, we can write

$$\mathrm{cov} \begin{bmatrix} \hat{S}_1 \\ \hat{S}_2 \end{bmatrix} = C \quad (13)$$

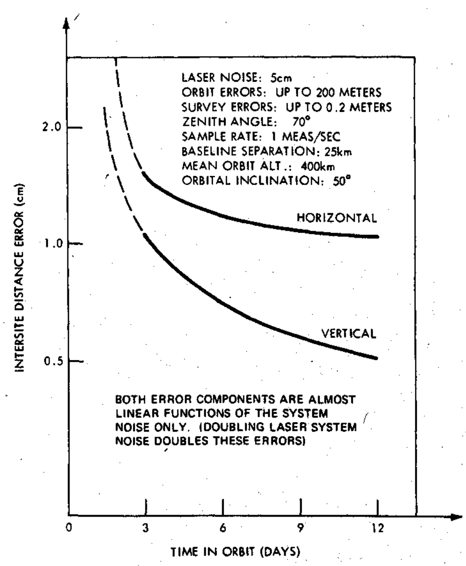

Fig. 4 Intersite distance errors vs time in orbit.

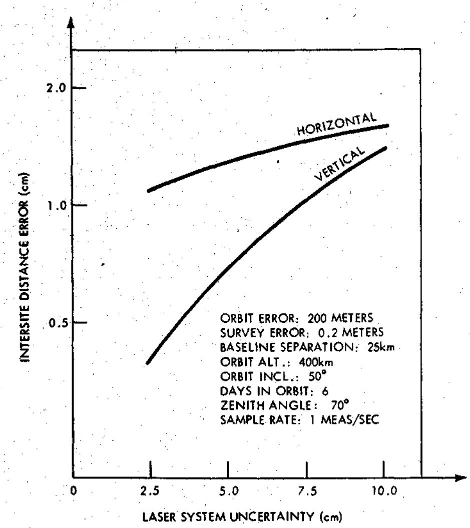

Fig. 5 Intersite distance errors vs laser system uncertainty.

where C is the 6×6 matrix in the lower-right-hand corner of the matrix described by the right side of Eq. (12). The estimate of the intersite distance is

$$\delta\hat{S} = \hat{S}_1 - \hat{S}_2 \quad (14)$$

Hence,

$$\mathrm{cov}(\delta\hat{S}) = DCD' \quad (15)$$

where

$$D = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix}$$

The elements of δS may be recoverable to a greater or lesser accuracy than the elements of S_1 or S_2, depending on the off-diagonal elements of C.

Error Analysis Results

A parametric study was performed to determine the accuracy with which intersite distances can be determined using the SPEAR concept. Equation (15) was assumed to describe the errors in intersite recovery. The results of the study are presented in Figs. 4 and 5. All assumptions concerning the source of errors and their magnitude are indicated on each figure. For these simulations, two ascending and two descending satellite passes were obtained per day. Satellite positions for each pass were obtained by propagating an epoch state over one orbital period.

Another set of simulations was performed to determine the effect of errors in satellite propagation on intersite distance recovery. The satellite force model was defined by a geopotential field described by a standard spherical harmonic expansion complete to degree and order four. The coefficients of the expansion were assumed to be uncertain to 5% of their nominal values and to be left unadjusted by the estimation process. Their uncertainties were permitted to impact the covariance matrix of δS. The results differed from the results shown in Figs. 4 and 5 by less than 0.5 cm. An analysis of the simulations shows that orbital errors affect the estimation of S_1 and S_2 in the same direction. Hence orbit errors tend to cancel in the determination of δS. This is, in part, an explanation for the exceptional accuracies displayed in Figs. 4 and 5.

Practical Applications

The SPEAR system can be developed as a payload for the Shuttle applications program. As a matter of fact, a breadboard model is under development at Goddard. Such a system can be utilized easily in detecting small relative variations of the Earth's upper crust in the 0.3- to 0.5-cm/yr range. Monitoring thus is possible of land subsidance as it occurs at coastal regions of Florida and Texas, as well as of construction sites such as dams and even large buildings. Dilatancy, the vertical uplift along seismic zones, can be followed further and thus studied as a precursory effect that takes place before many shallow foci earthquakes.[1] The San Andreas fault area and the Niigata area in Japan are examples.

Conclusions

This paper has shown that the SPEAR system can be used to determine intersite distances up to 15 km, depending only on the beam width or beam splitting of a spaceborne laser system within a six-day Shuttle mission to a precision 0.5 to 1.5 cm (assuming 50% cloud coverage). As anticipated, 1) survey errors (up to 0.25 m), 2) orbital errors (up to 200 m), and 3) range bias errors (many meters) play only a minor role, making the system potentially useful for many practical applications where very small relative motions are to be monitored. A further analysis shows that the intersite distance error increases with increased distance (about a factor of 3 for 100 km).

References

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